NON-ORTHOGONALITY RELATIONS BETWEEN COMPLEX-HYBRID-MODES: AN APPLICATION FOR THE LEAKY-WAVE ANALYSIS OF A LATERALLY-SHIELDED TOP-OPEN SUSPENDED MICROSTRIP LINE

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Abstract — **In this contribution, the orthogonality relations between complex modes are investigated to demonstrate the coupling effect which exists between TE^z and TM^z hybrid modes when power storage, loss or leakage is presented in the longitudinal direction of a transmission line. As an example, the Parallel Plate Waveguide (PPW) modes are used to study a laterally-shielded top-open microstrip line (LShMSW). The study of leaky-waves is a very interesting application of the novel orthogonality relations derived in this paper. This study is carried out using the correct and incorrect orthogonality formulations, and a comparison of results is made to check the validity of the proposed technique.**

I. INTRODUCTION

The orthogonality relations between propagation modes in guided-structures are well-known and are very important since the completeness property of the set of normal modes of a guided structure allows to expand any electromagnetic field inside the geometry [1], and allows the analysis of discontinuities and feed models in many devices such as microwave filters or multiplexers [2]-[3]. These relations have been studied by many authors, even for complex modes in planar transmission lines [4]-[6]. In these works, the inner product is often described by the following power coupling equation:

$$P_{mn} = \int_{x=0}^{a} \left[\vec{e}_m^{\,(p)}(x) \times \vec{h}_n^{\,(q)^*}(x) \right] \cdot \hat{z} \cdot \partial x \qquad (1)$$

Many other investigations have been reported to check the orthogonality relations of Complex Modes [4], leading to many interesting results. Of particular relevance are the conclusions presented in [4] which confirmed that the following inner product can be used to maintain the orthogonality property between general complex modes:

$$Q_{mn} = \int_{x=0}^{a} \left[\vec{e}_m^{\,(p)}(x) \times \vec{h}_n^{\,(q)}(x) \right] \cdot \hat{z} \cdot \partial x \qquad (2)$$

In this paper we show that none of these orthogonality relations are valid between TE^z and TM^z PPW hybrid modes in the context of the analysis of LShMSW, as shown in figure 1. This is due to the hybrid nature of these modes with respect to the radiation losses direction.

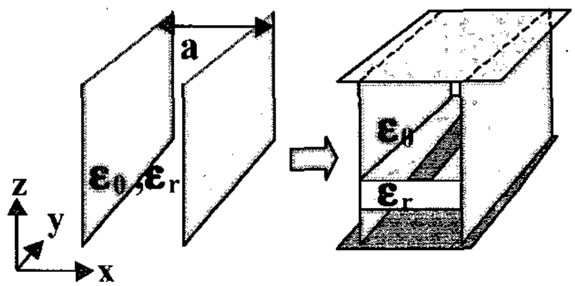

Fig. 1 Original PPW used to expand the fields of the final suspended microstrip structure.

In the development of the space-domain Green's functions for this problem, the fields are expanded by means of the modes of the PPW which supports the microstrip structure. An inner product must be used to compute the coupling between the different z-propagating PPW modes. It is demonstrated that TE^z and TM^z PPW modes are coupled when power storage, loss or leakage in the y-direction exists. Therefore, this coupling effect must be introduced in the formulation for a correct analysis. The paper develops a simple but rigorous model which takes into account all these coupling effects in the relevant Green's functions.

Results are presented for a leaky-wave mode of a laterally-shielded top-open suspended microstrip line. Since this type of mode is complex, the influence of the coupling between TE^z and TM^z hybrid modes in the final result is shown to be important. To check the validity of results, the induced currents on the strip are recovered using the magnetic field. It is demonstrated that the right currents are

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only recovered when the new proposed model is used, leading to wrong results if the coupling is not taken into account.

II. COMPLEX PPW MODES

The PPW field modes are needed to analyze the propagation characteristics of a laterally-shielded top-open suspended microstrip waveguide (LShMSW) as shown in figure 1. These are the customarily known PPW modes, but modified to allow for a propagating factor in the y axis. This propagating factor can in general be of complex nature with phase and amplitude parts:

$$k_y = \beta_y + j\alpha_y \tag{3}$$

These PPW modal fields can be analytically expressed by studying separately TE^z and TM^z plane-wave polarizations:

$$\vec{e}_m^{\,TE}(x,y,z) = \vec{e}_m^{\,TE}(x)\cdot e^{-jk_y\cdot y}\cdot e^{-jk_{zm}\cdot z} \tag{4}$$

$$\vec{e}_m^{\,TM}(x,y,z) = \vec{e}_m^{\,TM}(x)\cdot e^{-jk_y\cdot y}\cdot e^{-jk_{zm}\cdot z} \tag{5}$$

$$\vec{e}_m^{\,TE}(x) = \frac{-jk_y\cos(k_{xm}x)\hat{x} + k_{xm}\sin(k_{xm}x)\hat{y}}{N_m} \tag{6}$$

$$\vec{e}_m^{\,TM}(x) = \frac{+k_{xm}\cos(k_{xm}x)\hat{x} - jk_y\sin(k_{xm}x)\hat{y}}{N_m} \tag{7}$$

Where the modal propagation constant in the x-axis is determined by the lateral electric wall standing wave conditions:

$$k_{xm} = m\frac{\pi}{a}(rad/m) \tag{8}$$

Each mode is characterised by the modal index m (0,1..∞) and the polarization index p (TE^z or TM^z), and has a z-propagation factor in a medium with relative dielectric permittivity ε_r given by [10]:

$$k_{zm} = \sqrt{k_o^{\,2}\cdot\varepsilon_r - k_{xm}^{\,2} - k_y^{\,2}} \tag{9}$$

The magnetic field modal functions can be obtained with the next simple plane wave relation [10]:

$$\vec{h}_m^{\,(p)}(x,y,z) = \hat{z}\times\vec{e}_m^{\,(p)}(x,y,z) \tag{10}$$

Let us study the power orthogonality properties of these PPW modes. By applying (1) it can be obtained that:

$$P_{mn} = \begin{cases} 1 & ;\text{if } m=n \text{ and } p=q \\ 0 & ;\text{if } m\neq n \text{ and } p\neq q \\ C_m & ;\text{if } m=n \text{ and } p\neq q \end{cases} \tag{11}$$

Where it is found that a coupling Cm coefficient appears for modes with same space harmonic indexes m=n but different polarizations:

$$C_m = \frac{+k_{xm}\cdot 2\alpha_y}{|Nm|^2}\cdot\frac{a}{2}\delta_m \tag{12}$$

Where δ_n stands for Kronecker's delta function. This power coupling is nonzero only when k_y has an imaginary part, that is, when power storage, loss or leakage exists in the y-propagation direction for a LShMSW mode. This type of modes are important, since include evanescent (bellow cut-off and complex) modes, waves propagating in lossy materials, and leaky waves.

It might be thought that the inner product (2) described in [4] can decouple TE^z and TM^z polarizations for this type of LShMSW modes with an imaginary part in their propagation constant, but next results are obtained for our PPW modes:

$$Q_{mn} = \begin{cases} 1 & ;\text{if } m=n \text{ and } p=q \\ 0 & ;\text{if } m\neq n \text{ and } p\neq q \\ K_m & ;\text{if } m=n \text{ and } p\neq q \end{cases} \tag{13}$$

Where a new polarization-coupling coefficient is obtained:

$$K_m = \frac{-j2k_{xm}\cdot(\beta_y + j\alpha_y)}{N_m^{\,2}}\cdot\frac{a}{2}\delta_m \tag{14}$$

Not only the coupling phenomenon for complex modes still appears, but also it is extended for real modes, since now both the real and the imaginary parts of k_y are involved in Km. Therefore, the inner product (2) is not convenient for our purposes, since it complicates the formulation, and besides, the power coupling physical meaning of (1) is lost. This last inner product has a pure mathematical meaning, leading to a set of normal PPW modes which form an algebraic basis to expand the LShMSW fields.

III. GREEN'S FUNCTIONS FOR LATERALLY-SHIELDED TOP-OPEN SUSPENDED MICROSTRIP

The space-domain Green's functions for an electric source inside a multilayered-multiconductor structure have been developed in [7]-[8]. The electric and magnetic transverse fields produced by an elementary electric current inside the LShMSW can be expanded by the following PPW series:

$$\vec{E}t(x,y,z) = \sum_{p=1}^{2}\sum_{m=0}^{\infty} V_m^{(p)}(z)\cdot\vec{e}_m^{\,(p)}(x)\cdot e^{-jk_y} \tag{15}$$

$$\vec{H}t(x,y,z) = \sum_{p=1}^{2}\sum_{m=0}^{\infty} I_m^{(p)}(z)\cdot\vec{h}_m^{\,(p)}(x)\cdot e^{-jk_y} \tag{16}$$

The z-dependent scalar functions -Vm(z) and Im(z)- of above expressions can be obtained from Maxwell's transverse fields equations, leading to a set of modal equivalent transmission lines in the stratification z-axis. Following the same procedure as described in [7]-[8], but taking into account the coupling coefficient Cm between PPW modes, next equations are obtained:

$$\frac{\partial}{\partial z}\left[V_m^{TE}(z)+C_m\cdot V_m^{TM}(z)\right]=-j\cdot k_{zm}\cdot\left[Z_{0m}^{TE}\cdot I_m^{TE}(z)+C_m\cdot Z_{0m}^{TM}\cdot I_m^{TM}(z)\right] \quad (17)$$

$$\frac{\partial}{\partial z}\left[I_m^{TE}(z)+C_m\cdot I_m^{TM}(z)\right]=-j\cdot k_{zm}\cdot\left[\frac{V_m^{TE}(z)}{Z_{0m}^{TE}}+C_m\cdot\frac{V_m^{TM}(z)}{Z_{0m}^{TM}}\right]-\left[j_m^{TE}(z)+C_m\cdot j_m^{TM}(z)\right] \quad (18)$$

$$\frac{\partial}{\partial z}\left[V_m^{TM}(z)+C_m\cdot V_m^{TE}(z)\right]=-j\cdot k_{zm}\cdot\left[Z_{0m}^{TM}\cdot I_m^{TM}(z)+C_m\cdot Z_{0m}^{TE}\cdot I_m^{TE}(z)\right] \quad (19)$$

$$\frac{\partial}{\partial z}\left[I_m^{TM}(z)+C_m\cdot I_m^{TE}(z)\right]=-j\cdot k_{zm}\cdot\left[\frac{V_m^{TM}(z)}{Z_{0m}^{TM}}+C_m\cdot\frac{V_m^{TE}(z)}{Z_{0m}^{TE}}\right]-\left[j_m^{TM}(z)+C_m\cdot j_m^{TE}(z)\right] \quad (20)$$

The last four equations form a set of two coupled systems, each one corresponding to the coupled TE^z and TM^z equivalent transmission lines. In order to decouple them, we multiply (19) by Cm and subtract (17). Following the same procedure with (18) and (20) the following system of differential equations is obtained:

$$\frac{\partial}{\partial z}\left[V_m^{TE}(z)-C_m^2\cdot V_m^{TE}(z)\right]=-j\cdot k_{zm}\cdot Z_{0m}^{TE}\cdot\left[I_m^{TE}(z)-C_m^2\cdot I_m^{TE}(z)\right] \quad (21)$$

$$\frac{\partial}{\partial z}\left[I_m^{TE}(z)-C_m^2\cdot I_m^{TE}(z)\right]=-j\cdot\frac{k_{zm}}{Z_{0m}^{TE}}\cdot\left[V_m^{TE}(z)-C_m^2\cdot V_m^{TE}(z)\right]-\left[j_m^{TE}(z)-C_m^2\cdot j_m^{TE}(z)\right] \quad (22)$$

Proceeding in a similar way an analogous system is obtained but for the TM^z PPW modes. These two systems are decoupled in the sense that the voltage and current functions can be solved separately for each polarization:

$$V_{mTOTAL}^{p}(z)=V_m^{p}(z)-C_m^2\cdot V_m^{p}(z) \quad (23)$$

$$I_{mTOTAL}^{p}(z)=I_m^{p}(z)-C_m^2\cdot I_m^{p}(z) \quad (24)$$

However, the equivalent shunt current sources must be computed. Following the procedure described in [7]-[8], we obtain:

$$j_m^{(p)}=\frac{\vec{e}_m^{\,(p)*}(x')-C_m\cdot\vec{e}_m^{\,(q)*}(x')}{1-C_m^2}\quad\begin{cases}p=TE^z\rightarrow q=TM^z\\ p=TM^z\rightarrow q=TE^z\\ m=0,1,2..\end{cases} \quad (25)$$

As can be seen, the source of the p-equivalent transmission line suffers the coupling of the q-polarization source given by Cm. This situation can be represented by figure 2, where the equivalent modal TE^z and TM^z transmission lines are shown for the LShMSW.

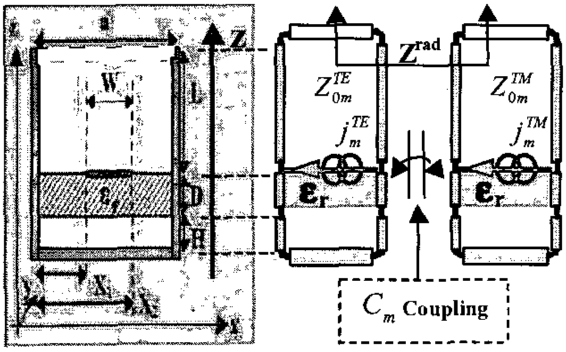

Fig. 2 Equivalent coupled transmission lines for LShMSW

With these equivalent modal-coupled networks, it is easy to find the equivalent voltage and current distribution along the z-axis by just applying classical Transmission Line Theory. Therefore, the electric and magnetic field amplitude functions involved in eq. (15-16) can thus be obtained.

IV. RESULTS FOR LEAKY WAVES

In order to investigate the coupling effect described in previous sections, a leaky-wave mode of the top-open laterally-shielded microstrip waveguide of figure 1 is studied. As it is known, this type of modes have a complex propagation constant in the longitudinal axis of the open waveguide, with a negative imaginary part due to the radiation losses. In figure 3, a leaky wave is searched by following the procedure described in [9]. The impedance of the top metallic wall of a completely shielded microstrip waveguide is changed gradually to the radiation top-open impedance Z^{rad} [10] of the final LShMSW in order to find the k_y solution from the original real guided-mode to the final complex solution of the leaky mode.

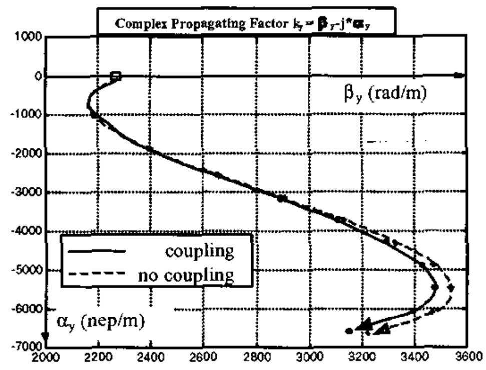

Fig. 3 Leaky mode search with and without coupling effects

As can be seen, two different solutions are found by taking into account or neglecting the polarization coupling. In order to know which solution is the valid one, the "currents induced" on the metal strip are recovered with the use of the boundary conditions for the tangential component of the magnetic field.

Basically, the currents on the metal strip are found by solving an EFIE homogeneous system. Once the system is solved, the "excitation currents" can be expanded, and from them the fields can be derived. Moreover, the "induced current" density in the strip can be computed from the magnetic fields boundary conditions and are then compared with the "excitation currents". In fig. 4, these two current densities are obtained for the case in which the coupling coefficients are neglected. As can be seen both currents are very different, and in addition, a non zero electric current appears outside the metal strip, which is physically impossible.

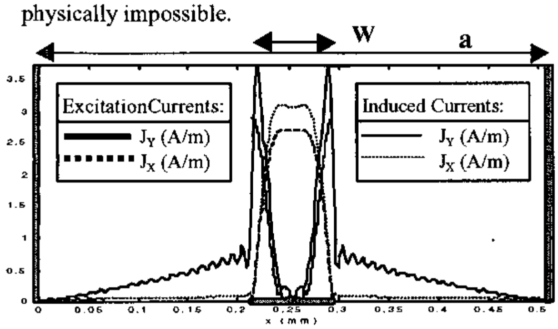

Fig. 4 Currents comparison neglecting coupling effects

Next, with the simple coupling model just derived in this paper, the results obtained are shown in fig. 5.

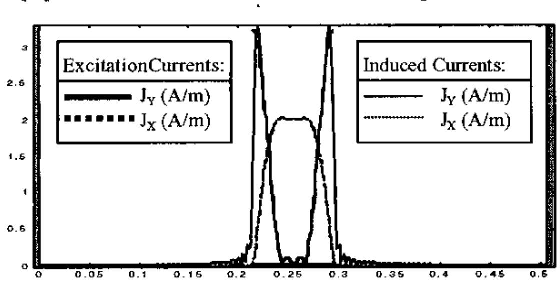

Fig. 5 Currents comparison with coupling effects

As can be seen, the "excitation currents" computed through the EFIE agree very well with the "induced currents" obtained with the magnetic field boundary condition. In particular, we can observe that the magnetic transverse field is continuous in the dielectric interface, leading to a zero electric current outside the metal strip, which confirm the validity of the method proposed.

V. CONCLUSIONS

In this contribution, it has been demonstrated that complex hybrid polarizations TE^z and TM^z are no orthogonal in the z-direction when storage, losses or power leakage exist in the y-direction. A suitable formulation has been developed in order to take into account these coupling effects, leading to a simple but precise equivalent model. This correction has been checked by studying a leaky-wave mode in a laterally-shielded top-open suspended microstrip waveguide. It is necessary to take into account the influence of the cross polarization coupling if accurate results are to be obtained when calculating the propagation characteristics of complex waves, such as evanescent, leaky or lossy modes.

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